



**Concursul Interdisciplinar „Vrănceanu-Procopiu”**

**Etapă județeană, județul Brașov**

**BAREM**

**Problema 1**

Considerăm  $a = r(\cos \alpha + i \sin \alpha)$ ,  $b = r(\cos \beta + i \sin \beta)$ ,  $c = r(\cos \gamma + i \sin \gamma)$

$$\frac{a}{b} + \frac{b}{c} + \frac{c}{a} = (\cos(\alpha - \beta) + \cos(\beta - \gamma) + \cos(\gamma - \alpha)) + i(\sin(\alpha - \beta) + \sin(\beta - \gamma) + \sin(\gamma - \alpha))$$

.....(2p)

$$\frac{a}{b} + \frac{b}{c} + \frac{c}{a} \in \mathbb{R} \iff \sin(\alpha - \beta) + \sin(\beta - \gamma) + \sin(\gamma - \alpha) = 0 \dots\dots\dots (2p)$$

$$\sin(\alpha - \beta) + \sin(\beta - \gamma) + \sin(\gamma - \alpha) = -4 \sin \frac{\alpha - \beta}{2} \sin \frac{\beta - \gamma}{2} \sin \frac{\gamma - \alpha}{2} \dots\dots\dots (2p)$$

$$\sin \frac{\alpha - \beta}{2} = 0 \iff \frac{\alpha - \beta}{2} = k\pi, k \in \mathbb{Z} \iff \alpha = \beta + 2k\pi \iff a = b \iff a - b = 0$$

.....(2p)

$$\frac{a}{b} + \frac{b}{c} + \frac{c}{a} \in \mathbb{R} \iff (a - b)(b - c)(c - a) = 0 \dots\dots\dots (1p)$$

Se acordă 1p oficiu.

**Problema 2**

$$\text{Notăm } t = \log_x y \Rightarrow \log_y x = \frac{1}{t} \dots\dots\dots (1p)$$

$$\text{Atunci } \log_x y + \log_y x \geq 2 \iff t + \frac{1}{t} \geq 2 \iff \frac{(t-1)^2}{t} \geq 0 \iff t > 0 \dots\dots\dots (3p)$$

$$t > 0 \iff \log_x y > 0 \iff x, y > 1 \text{ sau } 0 < x, y < 1 \dots\dots\dots (3p)$$

$$xy + 1 > x + y \iff (x - 1)(y - 1) > 0 \iff x, y > 1 \text{ sau } 0 < x, y < 1 \dots\dots (2p)$$

Se acordă 1p oficiu.